EE 302 FEEDBACK SYSTEMS

MIDTERM # 1

Mathematical Models:

- Differential Equations (for noobs)
- Transfer Function

$$G(s) = \frac{N(s)}{D(s)} = \frac{O(s)}{I(s)}$$

$N(s)$: roots are zeros
$O(s) \rightarrow$ output
$I(s) \rightarrow$ input
$D(s)$: roots are poles

- State-Space Representation:

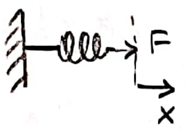
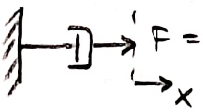
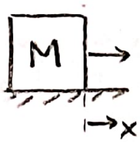
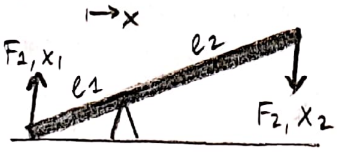
$$\dot{\underline{x}} = A\underline{x} + Bu$$
$$y = C\underline{x} + Du$$

$\underline{x}$: $n \times 1$ dimensional state vector
u: 1×1 input
y: 1×1 input

$A: n \times n$, $B: n \times 1$, $C: 1 \times n$, $D: 1 \times 1$

One can easily switch between these models. Main tool is $\mathcal{L}$.

Translational Mechanical Systems:

- Spring  $F = kx$
- Damper  $F = b\dot{x}$ (also viscous friction)
- Mass  $F = m\ddot{x}$
- Lever 

$F_1 l_1 = F_2 l_2$ (same torque)

$\frac{x_1}{l_1} = \frac{x_2}{l_2}$ (same work)

Transforming into electrical components:

$F \rightarrow i$
$\dot{x} \rightarrow v$

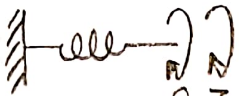
$b \rightarrow G = 1/R$ Damper becomes resistor (Solid surface is ground)

$k \rightarrow 1/L$ Spring becomes inductor

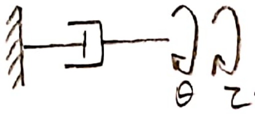
$M \rightarrow C$ Mass becomes capacitor

Lever becomes transformer

Rotational Systems

- Rotational Spring 

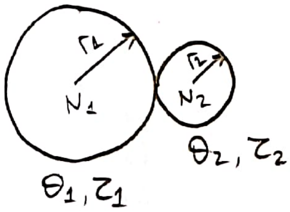
  $\theta \quad \tau = k\theta$

- Rotational Damper  $(\dot{\theta} = \omega)$

  $\theta \quad \tau = b\dot{\theta}$

- Rotational Inertia 

  $\theta \quad \tau = J\ddot{\theta}$

- Gears 

  $\theta_1 r_1 = \theta_2 r_2$ (same linear displacement)
  $\theta_1 N_1 = \theta_2 N_2$

  $\dfrac{\tau_1}{N_1} = \dfrac{\tau_2}{N_2}$ (same work)

Passage between rotational & translational systems:

$\tau = Fr \quad \& \quad x = \theta r$

DC Motors:

$\Phi = k_f i_f$

$\tau = k_M \Phi i_a \longrightarrow \tau = k_M k_f i_f i_a$ → $= k_\tau^f i_f$ for field controlled; → $= k_\tau^a i_a$ for armature controlled

$e_b = k_b \omega$

Block Diagram Simplification

- Eliminate feedback loops when possible (mind the signs)
- Make sure to preserve a function over a line same if that line is not eliminated
- You can always resort to using the error functions

Second Order Systems:

Standart form: $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

ω_n → undamped natural freq.

ξ → damping ratio

$\xi > 1$ overdamped response $s_{1,2} \in \mathbb{R}$, $s_1 \neq s_2$

$\xi = 1$ critically damped response $s_{1,2} \in \mathbb{R}$, $s_1 = s_2$

$0 < \xi < 1$ underdamped response $s_{1,2} \in \mathbb{C}$, $s_1 = s_2^*$

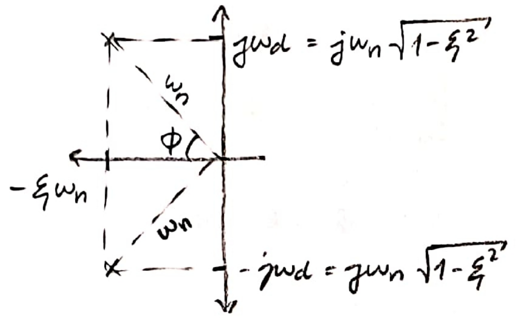
$\xi = 0$ undamped response $s_{1,2} \in \mathbb{C}$, $s_1 = s_2^*$, $Re\{s_{1,2}\} = 0$

Underdamped performance metrics:

- Maximum overshoot: $M_p = y(t_p) - y(\infty) = e^{-\pi/\tan\phi}$
- Peak time: $t_p = \dfrac{\pi}{\omega_d}$
- Rise time: $t_r = \dfrac{\pi - \phi}{\omega_d}$
- Settling time: (5%) $3/\xi\omega_n$

  (3%) $4/\xi\omega_n$

Above formulae make use of the pole locations & their implied properties:



$\cos\phi = \xi$

$\sin\phi = \sqrt{1-\xi^2}$

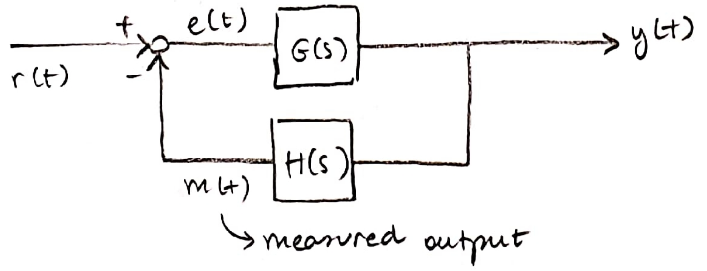
Overdamped Response: Pole(s) closest to $j\omega$-axis determine(s) the transient response

Higher Order Systems: The response can be approximated with a second order response given that

$$\max_{i \neq 1,2} \{ |\mathrm{Re}\{\sigma_i\}| \} > 3\,|\mathrm{Re}\{\sigma_{1,2}\}|$$

where $\sigma_{1,2}$ are the 2nd order approximation poles.

Steady State Error:



e(t): error

$$E(s) = \frac{1}{1 + G(s)H(s)} R(s)$$

Final Value Theorem: Let $F(s) = \mathcal{L}\{f(t)\}$. If $F(s)$ has all its poles in OLHP and/or at the origin, so if $\lim_{t\to\infty} f(t)$ exists, then

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Apply FVT to determine what happens to $e(t)$.

Unit step input:

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+G(s)H(s)} \frac{1}{s} = \lim_{s \to 0} \frac{1}{1+\underbrace{G(s)H(s)}_{K_p}}$$

K_p → static position error const.

Unit ramp input

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+G(s)H(s)} \frac{1}{s^2} = \dots = \frac{1}{K_v}$$

K_v → static velocity error const.

Unit acceleration input

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+G(s)H(s)} \frac{1}{s^3} = \dots = \frac{1}{K_a}$$

K_a → static acceleration error const.

BIBO Stability: A system is BIBO stable iff all its closed-loop poles lie in OLHP. So

$$Re\{s_i\} < 0 \quad \text{for} \quad D(s_i) = 0 \quad \forall i$$

D → also called characteristic polynomial

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MIDTERM #2

Routh's Stability Criterion: Given a polynomial $P(s)$ in the form of

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n \quad (a_0 > 0)$$

Construct the following array, namely the Routh array.

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & \dots \\
s^{n-1} & a_1 & a_3 & a_5 & \dots \\
s^{n-2} & b_1 & b_2 & b_3 & \dots \\
s^{n-3} & c_1 & c_2 & c_3 & \dots \\
\vdots & \vdots & \vdots & \vdots &
\end{array}$$

where

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad b_3 = \frac{a_0 a_6 - a_1 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \qquad \dots$$

Note that the construction is made by considering the above two rows, always the first column and the one left column. The intersection gives 4 elements:

$$\begin{array}{|ccc} k & & m \\ l & & n \\ & x & \end{array} \quad \Rightarrow \text{Then} \quad x = \frac{lm - kn}{l}$$

From this array, we make the following comments:

1) The polynomial $P(s)$ has all of its roots on the Open Left Half Plane if the elements of the first column are all positive

2) If there are negative elements in the first row, then the number of poles on the ORHP is equal to the number of sign changes in the first row.

There are certain difficulties we can encounter while constructing the Routh Array.

Difficulty #1: Obtaining 0 at the first row while there are other non-zero elements.

⇒ Solution #1: Replacing zero with $\epsilon > 0$, $\epsilon \approx 0$.

⇒ Solution #2: Replacing s with $\frac{1}{z}$ & solving for its zeros.

⇒ Solution #3: Multiplying P(s) with $(s+\alpha)$ where $\alpha > 0$.

Difficulty #2: Obtaining a completely zero row:

This means we have roots of equal magnitude lying radially opposite on the s-plane.

⇒ Solution: Go one level up & write an auxiliary polynomial. By magic, this auxiliary polynomial is a factor of P(S). This means we can replace its derivative's coefficients at the zero_row.

Note that if you encounter any difficulty, you definitely have roots with positive real parts.

Remark: If $D(s)$ has n roots with $Re\{\} > 0$, then $D(s-2)$ has n roots with $Re\{\} > 2$.

If $D(s-2)$ has roots with $Re\{\} > 0$, then $D(s)$ has n roots with $Re\{\} > -2$.

Root Locus Method: The root locus method is a root finding method for some (only) equation in the form of

$$D(s) + k\,N(s) = 0.$$

The roots of the equation are found as the value K is varied from 0 to infinity.

We use this method in finding the pole locations of a closed loop transfer function, given the open loop transfer function. If the open loop transfer function is of the form

$$G(s)\,H(s) = K.\frac{N(s)}{D(s)}$$

Then the closed loop poles satisfy the equation

$$1 + G(s)H(s) = 0 \Rightarrow 1 + k\frac{N(s)}{D(s)} = 0 \Rightarrow D(s) + k.N(s) = 0$$

which concurs with the method's required format. BUT, it is a general method!

The rules of root locus are as follows: ($\deg(D(s)) = n$, $\deg(N(s)) = m$)

1) The root locus plot starts from the open loop poles & ends at either infinity or at open loop zeros.

2) The number of branches on a root locus is $\max\{m,n\}$

3) The number of asymptotes is $|m-n|$

4) A root locus plot is symmetric wrt. real axis.

5) Angle between asymptotes is $\dfrac{2\pi}{|m-n|}$

6) Angle of the first asymptote is $\dfrac{\pi}{|m-n|}$

7) The intersection of the asymptotes is at

$$\sigma_0 = \frac{\sum p_i - \sum z_i}{n-m}$$

where p_i are the roots of $D(s)$ & z_i are those of $N(s)$.

8) A point on the real axis is included in the root locus if it has an odd number of poles & zeros on its right.

In applying these, all we're doing in reality is solving for the angle & magnitude conditions of the characteristic equation:

$$1 + G(s)H(s) = 0 \Rightarrow G(s)H(s) = -1 \quad \begin{cases} \angle G(s) + \angle H(s) = (2k+1)\pi \\ |G(s)H(s)| = 1 \end{cases}$$

Break-away/in points: At such points, say s_0, s_0 is a double root, so it is also a root of $(D(s)+kN(s))'$ polynomial. Then at such points:

$$D'(s)N(s) - D(s)N'(s) = 0$$

If a solution to this equation is valid, its corresponding k value must be positive, or at least non-negative.

$$k = -\frac{D(s)}{N(s)}$$

The intersection with the $j\omega$-axis can be found by inserting $s = j\omega$ & solving for k & ω by

$$Re\{\} = 0 \quad \& \quad Im\{\} = 0$$

Or you can just apply the Routh Hurwitz, & look for a zero row.

Nyquist Stability Criterion: First, back to Cauchy:

Cauchy's Principle of Argument: Given an arbitrary closed path Γ_s and a rational function $T(s) = A(s)/B(s)$, the encirclement of the origin by $\Gamma_{T(s)}$ is

$\circlearrowright$ = +1 encirclement

$$N_0 = \#\text{ zeros of } T(s) \text{ inside } \Gamma_s - \#\text{ poles of } T(s) \text{ inside } \Gamma_s$$

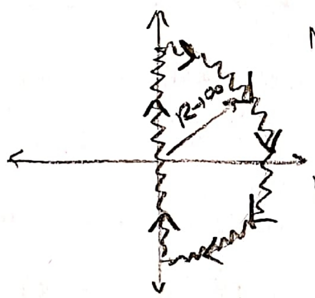
$\circlearrowleft$ = -1 encirclement

or equivalently,

$$N_0 = \#\text{ of roots of } A(s) \text{ inside } \Gamma_s - \#\text{ roots of } B(s) \text{ inside } \Gamma_s.$$

Here, the contour Γ_s must not cross over any poles or zeros of $T(s)$.

To be able to use this theorem to our benefit, we select a very specific Γ_s: Nyquist Path.



Note that this Nyquist Path can be modified to move around any poles or zeros existing on the $j\omega$-axis.

Now, let the open loop transfer function be of the form

$$K\frac{N(s)}{D(s)}$$

Method:

- Sketch $\Gamma_{\frac{N(s)}{D(s)}}$ on the s-plane
- Check the encirclement of $-\frac{1}{K}$: $N_{-\frac{1}{K}}$, which gives

$$N_{-\frac{1}{K}} = \#\text{ closed loop poles inside } \Gamma_s \text{ (i.e. ORHP)} - \#\text{ open loop poles inside } \Gamma_s$$

Why do we do that?

$$G(s)H(s) = K\frac{N(s)}{D(s)} \Rightarrow 1 + G(s)H(s) = 1 + K\frac{N(s)}{D(s)} = 0$$

$$K\left(\frac{1}{K} + \frac{N(s)}{D(s)}\right) = 0 \underset{(K\neq 0)}{\Rightarrow} \frac{D(s)+KN(s)}{KD(s)} = 0 \qquad \begin{matrix} \longrightarrow A(s) \\ \longrightarrow B(s) \end{matrix}$$

Roots of $A(s)$ here are the closed loop poles & those of $B(s)$ are the open loop poles.

Then I am actually looking for the encirclement of the origin by the function

$$\frac{1}{k} + \frac{N(s)}{D(s)} = P(s)$$

Now if this function encircles the origin N many times, $P(s) - \frac{1}{k}$ encircles the $\frac{-1}{k}$ N many times, since $P(s) - \frac{1}{k}$ is just shifting the $T_{P(s)}$ to the left by the amount of $\frac{1}{k}$, so instead of the encirclement of the origin by $T_{P(s)} = T_{\frac{1}{k} + \frac{N(s)}{D(s)}}$, I can just check the encirclement of $-\frac{1}{k}$ by $T_{P(s) - \frac{1}{k}} = T_{\frac{N(s)}{D(s)}}$, which are the same thing!

The most significant part of the $T_{\frac{N(s)}{D(s)}}$ is obtained as we are moving on the $j\omega$ axis, starting from the origin. This gives us the polar plot. The polar plot is a more compact representation of the Bode plots: We have Bode plots for magnitude and angle, the polar plot combines them on the s-plane. The round part, i.e. the $R \to \infty$, $Re^{j\theta}$, $\theta \in [\pi/2, -\pi/2]$ part, corresponds to moving on the origin, which is not significant. Conversely, small turns made around the poles and/or zeros on the $j\omega$ axis with the modified Nyquist path produces infinite-radius movements.

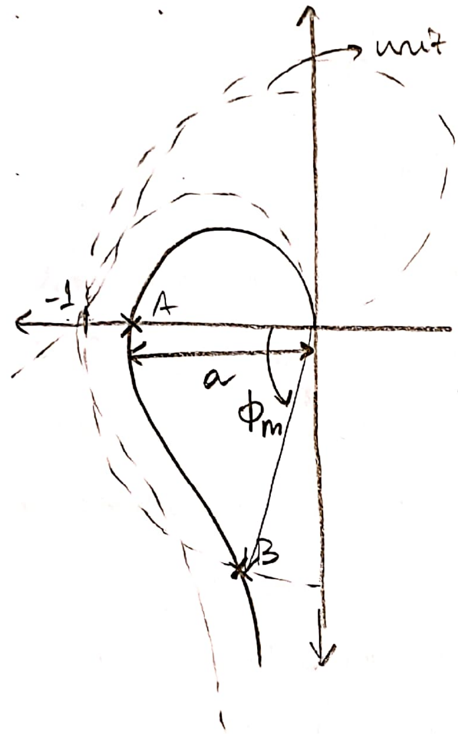
Note that the tiny movements around the origin, i.e. $s = \epsilon e^{j\theta}$, $\epsilon \approx 0$, $\theta : 0 \nearrow 90°$ can be used to identify the system type, because the only terms surviving at the denominator are the $s = \epsilon e^{j\theta}$ ones, and so the amount of turn indicates the system type. If the function makes a $+90°$ turn, then a system of type N makes an infinite radius turn of $-N.90°$ turn. If it is a $+180°$ turn, say that $\theta : -90° \nearrow 90°$, then the resulting Nyquist plot makes a turn of $-N.180°$.

Relative Stability: We have two aspects of relative stability:

- Gain margin: How much you can increase the OL gain before the system becomes unstable.

- Phase margin: How much phase you can add to the system before the system becomes unstable

Say the polar plot is like the one below:



At point A: $\omega = \omega_p \Rightarrow$ phase crossover frequency

gain margin $= \frac{1}{a}$

because multiplying the mapping by $\frac{1}{a}$ would result in crossing over the -1 point, which means $G(j\omega)H(j\omega) = -1$ and so $G(j\omega)H(j\omega) + 1 = 0$, producing a pole at ω_p & making the system unstable. (encircling -1)

At point B: $\omega = \omega_g \Rightarrow$ gain crossover frequency

phase margin $= \phi_m$

because multiplying the mapping by $e^{-j\phi_m}$ would result in -1 being encircled, hence an unstable system.

Why does encircling -1 yields unstability?

The mapping we have at hand now is $K\frac{N(s)}{D(s)}$. Remember that we are originally looking for a solution to $G(s)H(s) + 1 = 0$ equation for the closed loop poles, and we assumed that $G(s)H(s) = K\frac{N(s)}{D(s)}$. So we are solving for $1 + K\frac{N(s)}{D(s)} = 0$. But instead of checking the encirclement of the origin by $T_{1+K\frac{N(s)}{D(s)}}$, we can check the encirclement of -1 by $T_{K\frac{N(s)}{D(s)}}$, which is the same thing. Here K is limited by both its magnitude & its angle, hence the gain & phase margins.

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FINAL

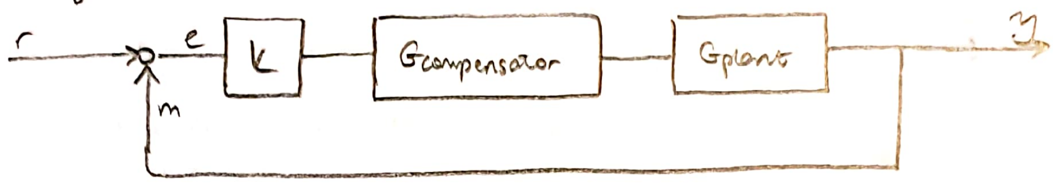
PHASE LEAD & LAG COMPENSATORS

The main ideas:

Phase lead: Add phase at the gain cross-over frequency

Phase lag: Drop gain at the gain crossover frequency

Let the system be as follows:



Phase Lead Compensator Design Procedure:

$$G_{compensator}(s) = \frac{1+Ts}{1+aTs} \qquad T>0 \qquad 0<a<1$$

0) Obtain the Bode Plot for G(s)

1) Find K to satisfy ess requirement.

2) Combine Gplant (s) & K and call it Guncompensated (s)

3) Obtain the Bode plot for Guncompensated (s)

4) Find the gain crossover frequency w_g and the phase margin of the uncompensated system.

$$|G_{uncomp.}(jw_g)| = 1$$

$$p.m. = \angle G_{uncomp}(jw_g) + 180^\circ$$

5) Find the increment ϕ_{max} to be added to the system via the compensator and compute a

$$\text{Required p.m.} + 5^\circ \sim 12^\circ \text{ margin} = \phi_{max} + \text{Uncomp. p.m.}$$

$$a = \frac{1-\sin(\phi_{max})}{1+\sin(\phi_{max})}$$

6) Place the peak phase ϕ_{max} at the new gain crossover frequency $w_{g,new}$ and find T.

$$|G_{uncomp.}(jw_{g,new})| = \sqrt{a} = 10\log\frac{1}{a} \text{ in dB.}$$

$$w_{g,new} = \frac{1}{\sqrt{a}\,T}$$

Phase Lag Compensator Design Procedure:

$$G_{compensator} = \frac{1+Ts}{1+BTs} \qquad T>0 \qquad B>1$$

0) Obtain the Bode Plot of the system.

1) Find k to satisfy e_{ss} requirement

2) Combine k & $G_{plant}(s)$ to obtain $G_{uncompensated}(s)$

3) Obtain the Bode Plot of $G_{uncomp.}(s)$.

4) Find the new gain crossover frequency by

$$\angle G_{uncomp}(j\omega_{g,new}) = -180^\circ + \text{Required p.m.} + 5^\circ \sim 12^\circ \text{ margin}$$

5) Find B such that

$$|G_{uncomp}(j\omega_{g,new})| = B$$

6) Find T such that the larger corner frequency $\frac{1}{T}$ is one decade below $\omega_{g,new}$:

$$\frac{10}{T} = \omega_{g,new}$$

Limitations of Compensators:

Phase Lead: Impossible to add more than 60° phase due to circuit limitations.

Phase Lag: If the phase response of the original system does not exhibit the required phase margin, phase lag compensator becomes useless.

Bode Plots Reminders:

$K \longrightarrow 20 \log K$ constant line

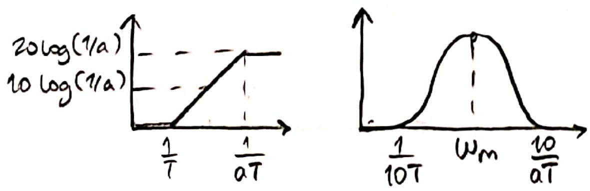
$s \longrightarrow +20\text{dB/dec}$ line, passing through $(\omega=0,\ dB=0)$ $\quad +90^\circ$

$\frac{1}{s} \longrightarrow -20\text{ dB/dec}$ line, passing through $(\omega=0,\ dB=0)$ $\quad -90^\circ$

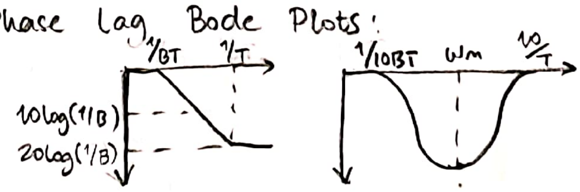
$(1+sT) \longrightarrow +20\text{ dB/dec}$ after passing $\omega = \frac{1}{T}$ $\quad +45\text{ dB/dec}$

$\frac{1}{1+sT} \longrightarrow -20\text{ dB/dec}$ after passing $\omega = \frac{1}{T}$ $\quad -45\text{ dB/dec}$

Phase Lead Bode Plots:



Phase Lag Bode Plots:



STATE SPACE REPRESENTATION

- State Space Representation:

$$\dot{x} = Ax + Bu \quad \Rightarrow \text{dynamics equation}$$

$$y = Cx \quad \Rightarrow \text{measurement equation}$$ (omit Du for simplicity)

where

$x \in \mathbb{R}^{n\times 1}$ n-dimensional column vector

$A \in \mathbb{R}^{n\times n}$ $n\times n$ matrix

$B \in \mathbb{R}^{n\times 1}$ n-dimensional column vector

$C \in \mathbb{R}^{1\times n}$ n-dimensional <u>row</u> vector

- The transfer function is found by the following derived formula:

$$sX = AX + BU \quad \& \quad Y = CX$$

$$(sI - A)X = BU$$

$$X = (sI - A)^{-1}BU$$

$$Y = \underbrace{C(sI-A)^{-1}B}_{T(s) \text{ the transfer function}}U$$

Changing the state vector by means of $x_{new} = P_{n\times n}\, x_{old}$ does <u>not</u> change the transfer function.

Diagonal Canonical Form

- Let $T(s) = \dfrac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n} = \sum_{i=1}^{n} \alpha_i \dfrac{U(s)}{s - p_i}$

IF $T(s)$ HAS NO REPEATED POLES, namely p_i's are unique, call your states as

$$x_i(s) \triangleq \frac{U(s)}{s - p_i} \quad \Rightarrow \quad sX_i(s) = p_i X_i(s) + U(s)$$

So

$$A_{DCF} = \mathrm{diag}(p_1, p_2, \dots, p_i, \dots p_n)$$

$$B_{DCF} = [1 \;\; 1 \;\; 1 \dots \;\; 1]^T$$

$$C_{DCF} = [\alpha_1 \;\; \alpha_2 \; \dots \; \alpha_i \; \dots \; \alpha_n]$$

Controllable Canonical Form:

Let $T(s) = \dfrac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}{s^n + a_1 s^{n-1} + \dots + a_n}$, Name your poles directly

from this form and write

$$A_{CCF} = \begin{bmatrix} 0 & 1 & & \cdots & 0 \\ \vdots & & 1 & & \vdots \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & \cdots & & 1 \\ -a_n & \cdots & -a_i & \cdots & -a_1 \end{bmatrix}$$

$$B_{CCF} = [0 \dots 0 \; 1]^T$$

$$C_{CCF} = [b_n \dots b_i \dots b_1]$$

Observable Canonical Form:

Notice that $T(s)^T = T(s) = B^T (sI - A^T)^{-1} C^T$. Apply this to the Controllable Canonical Form Matrices:

$$A_{OCF} = A_{CCF}^T = \begin{bmatrix} 0 & \cdots & & 0 & -a_n \\ 1 & & \cdots & 0 & -a_{n-1} \\ & 1 & & \vdots & \vdots \\ \vdots & & \ddots & & -a_i \\ & & & 1 & \\ 0 & \cdots & & 1 & -a_1 \end{bmatrix}$$

$$B_{OCF} = C_{CCF}^T = [b_n \dots b_i \dots b_1]^T$$

$$C_{OCF} = B_{CCF}^T = [0 \dots 0 \; 1]$$

Stability Analysis for State Space Representations

Notice that $\det(sI - A)$ is at the denominator of the transfer function. Although not all roots of $\det(sI-A)$ may show up as poles, poles are selected from these roots. So, if all the roots of $\det(sI-A)$ namely the eigenvalues of the matrix A, have negative real parts, then the system is assuredly stable.

$$T(s) = C(sI-A)^{-1}B = \frac{1}{\det(sI-A)} \cdot C \cdot \operatorname{Adj}(sI-A) \cdot B$$

Controllability:

An LTI system in state space form is said to be controllable if it is possible to transfer the state vector from any initial state $x(0)$ to any other state $x(t)$ in a finite amount of time by using a suitable input $u(t)$.

The controllability matrix is defined as

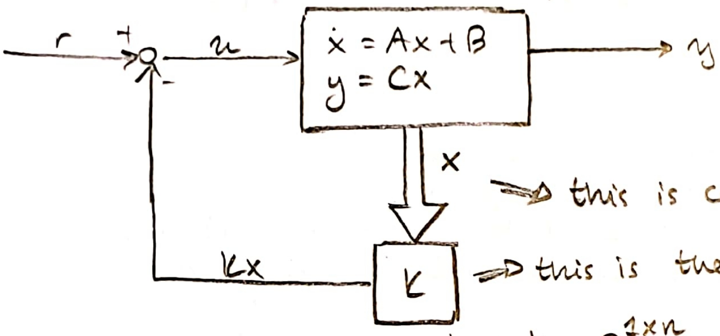
$$Q = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

A system is controllable if the matrix Q is invertible.

$$\det(Q) \neq 0$$

State Feedback:

Let's redefine our feedback loop:

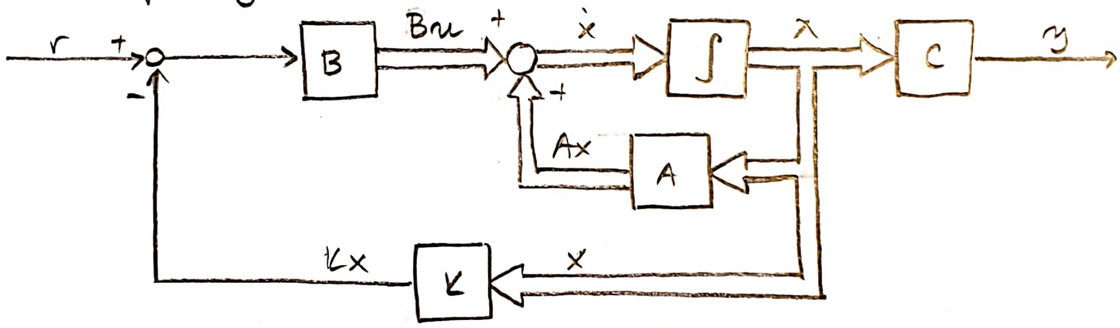


→ this is called "state feedback"

→ this is the "feedback gain"

→ $K \in \mathbb{R}^{1\times n}$ n-dimensional row vector

More explicitly:



Then we can say that $\begin{Bmatrix} \dot{x} = Ax + Bu \\ y = Cx \end{Bmatrix}$ is the new open loop system.

The closed loop system is then

$$\dot{x} = Ax + B(r - Kx) = \underbrace{(A - BK)}_{A_{cl}}x + Br \quad \& \quad y = Cx$$

$$\begin{Bmatrix} \dot{x} = (A-BK)x + Br \\ y = Cx \end{Bmatrix}$$

The open loop poles come from eigen values of A:

$$\det(sI - A) = 0$$

The closed loop poles come from eigenvalues of A_{cl}

$$\det(sI - A_{cl}) = \det(sI - A + BK) = 0$$

Pole Placement:

Our aim is to adjust $K = [k_1 \ k_2 \ --- \ k_n]$ matrix so that our closed loop poles are exactly wherever we want.

Just equate the desired characteristic equation to $\det(sI - A_{cl})$

The good thing is, if you start with a controllable canonical form, your closed loop system will again be in controllable canonical form, so you don't have to work with determinant and instead can just write the transfer function.

Observability:

An LTI system in state space form is said to be observable if it is possible to calculate the initial state $x(0)$ exactly from the knowledge of the output $y(t)$ and the input $u(t)$ over a finite time interval for any $x(0)$ and input u.

The observability matrix W is defined as

$$W = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

A system is observable if the matrix W is invertible

$$\det(W) \neq 0$$

Obvious remarks:

A system in CCF is always controllable

A system in OCF is always observable.